OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4752

Concepts for Advanced Mathematics (C2)

Monday **16 JANUARY 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 5 printed pages and 3 blank pages.

Section A (36 marks)

1 Given that $140° = k\pi$ radians, find the exact value of k. [2]

2 Find the numerical value of $\sum_{k=2}^{5} k^3$. [2]

3

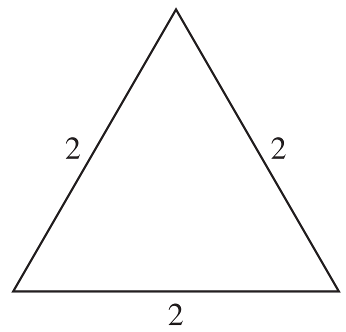


Fig. 3

Beginning with the triangle shown in Fig. 3, prove that $\sin 60° = \frac{\sqrt{3}}{2}$. [3]

4

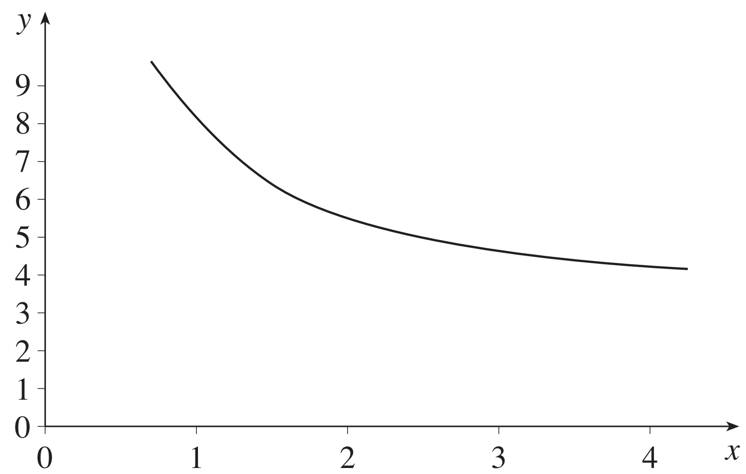


Fig. 4

Fig. 4 shows a curve which passes through the points shown in the following table.

x	1	1.5	2	2.5	3	3.5	4
y	8.2	6.4	5.5	5.0	4.7	4.4	4.2

Use the trapezium rule with 6 strips to estimate the area of the region bounded by the curve, the lines $x = 1$ and $x = 4$, and the x-axis.

State, with a reason, whether the trapezium rule gives an overestimate or an underestimate of the area of this region. [5]

5 **(i)** Sketch the graph of $y = \tan x$ for $0° \leqslant x \leqslant 360°$. [2]

(ii) Solve the equation $4\sin x = 3\cos x$ for $0° \leqslant x \leqslant 360°$. [3]

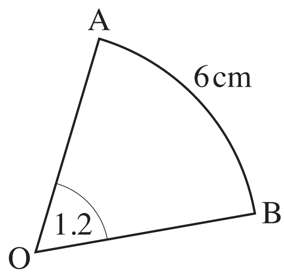
6 A curve has gradient given by $\dfrac{dy}{dx} = x^2 - 6x + 9$. Find $\dfrac{d^2y}{dx^2}$.

Show that the curve has a stationary point of inflection when $x = 3$. [4]

7 In Fig. 7, A and B are points on the circumference of a circle with centre O.

Angle AOB = 1.2 radians.

The arc length AB is 6 cm.



Not to scale

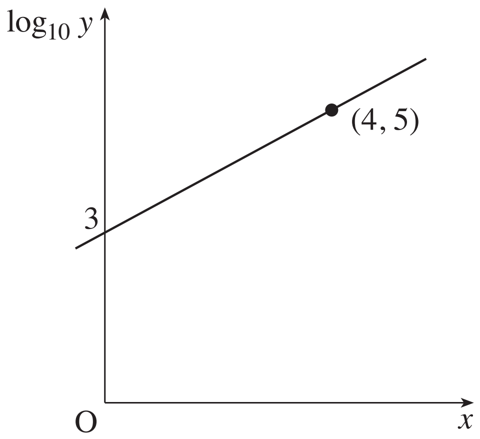
Fig. 7

(i) Calculate the radius of the circle. [2]

(ii) Calculate the length of the chord AB. [3]

8 Find $\displaystyle\int \left(x^{\frac{1}{2}} + \frac{6}{x^3} \right) dx$. [5]

9



Not to scale

Fig. 9

The graph of $\log_{10} y$ against x is a straight line as shown in Fig. 9.

(i) Find the equation for $\log_{10} y$ in terms of x. [3]

(ii) Find the equation for y in terms of x. [2]

Section B (36 marks)

10 The equation of a curve is $y = 7 + 6x - x^2$.

(i) Use calculus to find the coordinates of the turning point on this curve.

Find also the coordinates of the points of intersection of this curve with the axes, and sketch the curve. [8]

(ii) Find $\int_1^5 (7 + 6x - x^2)\,dx$, showing your working. [3]

(iii) The curve and the line $y = 12$ intersect at $(1, 12)$ and $(5, 12)$. Using your answer to part **(ii)**, find the area of the finite region between the curve and the line $y = 12$. [1]

11

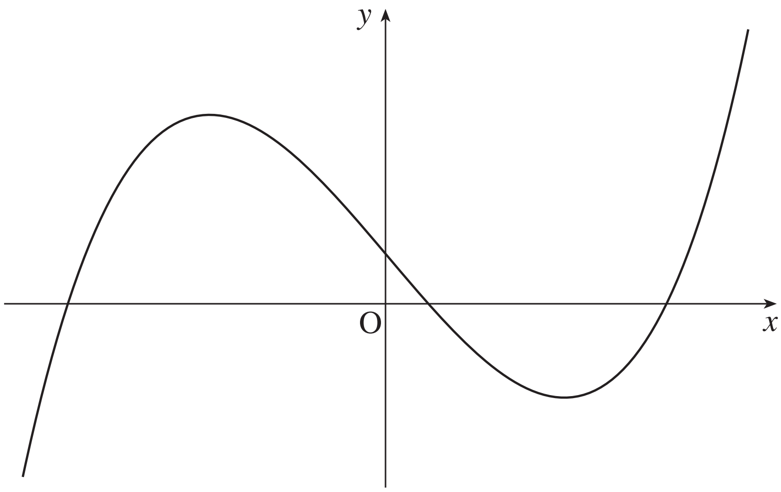


Fig. 11

The equation of the curve shown in Fig. 11 is $y = x^3 - 6x + 2$.

(i) Find $\frac{dy}{dx}$. [2]

(ii) Find, in exact form, the range of values of x for which $x^3 - 6x + 2$ is a decreasing function. [3]

(iii) Find the equation of the tangent to the curve at the point $(-1, 7)$.

Find also the coordinates of the point where this tangent crosses the curve again. [6]

12 **(i)** Granny gives Simon £5 on his 1st birthday. On each successive birthday, she gives him £2 more than she did the previous year.

(A) How much does she give him on his 10th birthday? [2]

(B) How old is he when she gives him £51? [2]

(C) How much has she given him **in total** when he has had his 20th birthday present? [2]

(ii) Grandpa gives Simon £5 on his 1st birthday and increases the amount by 10% each year.

(A) How much does he give Simon on his 10th birthday? [2]

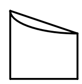
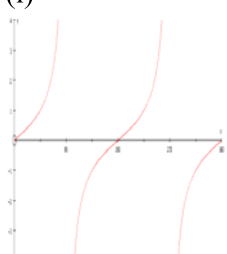
(B) Simon first gets a present of over £50 from Grandpa on his nth birthday. Show that

$$n > \frac{1}{\log_{10} 1.1} + 1.$$

Find the value of n. [5]

Mark Scheme 4752
January 2006

Section A

1	7/9 or 140/180 o.e.	2	B1 for 180° = π rad o.e. or 0.78 or other approximations	2
2	224	2	M1 for $2^3 + 3^3 + 4^3 + 5^3$	2
3	triangle divided into 2 rt angled tris √3 and 1 indicated 60 indicated	H1 S1 A1		3
4	16.1  overestimate + expn eg sketch	4 1	M3 for ¼{8.2 + 4.2 + 2 (6.4 + 5.5 + 5 + 4.7 + 4.4)} M2 for one slip/error M1 for two slips/errors	5
5	(i)  tan x = ¾ (ii) 36.8 to 36.9 and 216.8 to 216.9	2 M1 A1A1	no numbers required on axes unless more branches shown. G1 for a correct first sweep Allow 37, 217	5
6	$y'' = 2x - 6$ $y'' = 0$ at $x = 3$ $y' = 0$ at $x = 3$ showing y' does not change sign	B1 B1 B1 E1	or that y'' changes sign	4
7	(i) 5 (ii) 5.646... to 2 sf or more	2 3	M1 for 6 = 1.2r M2 for 2 x 5x sin 0.6 or $\sqrt{(5^2 + 5^2 - 2.5.5.\cos 1.2)}$ or 5 sin 1.2/sin 0.971 M1 for these methods with 1 error	5
8	$\frac{2}{3}x^{\frac{3}{2}} - 3x^{-2} + c$ o.e.	5	1 for each element	5
9	(i) $\log_{10} y = 0.5x + 3$ (ii) $y = 10^{0.5x+3}$ isw	B3 2	B1 for each term scored in either part o.e. e.g. $y = 1000 \times 10^{\sqrt{x}}$	5

Section B

10	**i**	$y' = 6 - 2x$ $y' = 0$ used $x = 3$ $y = 16$	M1 M1 A1 A1	condone one error	
		(0, 7) (–1, 0) and (7,0) found or marked on graph	3	1 each	
		sketch of correct shape	1	must reach pos. y - axis	8
	ii	58.6 to 58.7	3 M1	B1 for $7x + 3x^2 - x^3/3$ [their value at 5] – [their value at 1] dependent on integration attempted	3
	iii	using his (ii) and 48	1		1
11	**i**	$3x^2 - 6$	2	1 if one error	2
	ii	$-\sqrt{2} < x < \sqrt{2}$	3	M1 for using their $y' = 0$ B1 f.t. for both roots found	3
	iii	subst $x = -1$ in their y' [=–3] $y = 7$ when $x = -1$ $y + 3x = 4$	B1 M1 A1	f.t. f.t. 3 terms	
		$x^3 - 6x + 2 = -3x + 4$ (2, –2) c.a.o.	M1 A1,A1	f.t.	6
12	**i**	A 23	2	M1 for 5, 7, 9 etc or AP with $a = 5$, $d = 2$	2
		B 24	2	M1 for $51 = 5 + 2(n - 1)$ o.e.	2
		C 480	2	M1 for attempted use of sum of AP formula eg 20/2[10+19×2]	2
	ii	A 11.78 – 11.80	2		
		B $5 \times 1.1^{n-1} > 50$ $1.1^{n-1} > 10$ $(n - 1) \log 1.1 > 1$ $n - 1 > 1/\log 1.1$	B1 B1 L1 A1	Or other step towards completion (NB answer given)	
		n = 26	1	independent	

4752: C2 Concepts for Advanced Mathematics

General Comments

The paper was well received and no-one seemed to be short of time. All questions were accessible to at least half of the candidates. There was the usual number of excellent scripts and rather fewer ones scoring less than 15 marks.

Comments on Individual Questions

Section A

1) Most candidates knew how to convert degrees to radians. The request to find the value of k confused many and they submitted the answers $^{7\pi}/_{9}$ or 2.44. The word "exact" in the question warns against using calculators, k = 0.778 was penalised. Weak candidates simply solved our "equation" 140 = kπ to get 44.6.

2) Successful candidates made a simple interpretation of the sigma notation; many spoiled their efforts by including 6 cubed. They must have thought that summing to 5 implied 5 terms. Weaker candidates quoted inappropriate formulae from work with APs or GPs. Interestingly many candidates studying for higher papers used the formula for the sum of cubes, and they used it correctly except that not one of them noticed that it did not start at 1, they all got 225.

3) The Examiners did want to see a diagram for this work, and most candidates provided one. We wanted to see Pythagoras's Theorem applied to the half triangle and the relevant figures identified on their diagram. Just a few argued with the cosine rule (no diagram needed) to get cos 60 = ½, $\sin^2 60$ = ¾ and hence the result.

4) Candidates worked well here and, avoiding all the opportunities to make a slip, successfully found the required area. Many did make slips of course and some got the strip width wrong and some failed to use the two sets of brackets correctly. Most correctly identified the overestimate with more or less convincing reasons. Some made it clear on their diagrams that their trapezia were in fact rectangles, this was penalized.

5) (i) The sketches were often excellent.

(ii) Those who separated the trig. functions and the numbers were usually successful. Unfortunately a very large fraction of the entry divided LHS by RHS to get $^{4}/_{3}$ tan *x*. If they had left the RHS as 1 they had a chance, but it was invariably left blank or 0. The candidates were stuck with $^{4}/_{3}$ and they had no good ideas as to how to handle it.

6) Most correctly found *y*" and showed that *y*"(3) = 0. Some also showed that *y*′(3) = 0 and so three of the four marks were earned. The only completely successful attempts showed a before and after test on *y*′. No-one said that *y*" was 0 and was changing sign.

7) (i) This was usually correct.

(ii) The three main methods used were (a) cosine rule on triangle OAB (b) calculate the base angles of OAB and use the sine rule (c) bisect the triangle and use 2 x 5 sin 0.6.

8) Apart from very weak candidates who differentiated or integrated x^3 in situ this was a good source of marks for most. The fifth mark was for showing the arbitrary constant somewhere.

9) The dependent variable being log *y* confused many candidates; they assumed that all the given coordinates should also be logged. There were two marks for arriving at $0.5x + 3$ whatever they called it. Many good candidates coped very well and had no problem converting their part (i) equation to the other form. The commonest error was to convert $\log y = 0.5x + 3$ to $y = 10^{0.5x} + 10^3$.

Section B

10) The whole of this question was very well done by many and full marks were common.
In (i) there was a penalty for those who did not use calculus, they were asked to. Some completed the square and even "sum of the roots = -b/a" was seen.
Part (ii) was often fully correct and in part (iii) many could see a rectangle of area 48 if they had produced a decent sketch in part (i).

11) (i) This part was usually correct.

(ii) Part (ii) was often not attempted. Many of those who did work here suffered from very poor arithmetic and algebra; those who got as far as $x^2 = 2$ then said $x = \sqrt{2}$ or ± 2 or ± 1. Good candidates had no trouble finding the required range.

(iii) In part (iii) many knew how to find the equation of the tangent but again suffered from carelessness. Just a few thought that the gradient at (-1,7) was $3x^2 - 6$ and many more than those converted their gradient of -3 to 1/3, just because "it's always tested". Solving the simultaneous equations usually scored a mark for the method but few battled through to the correct coordinates.

12) The candidates were well versed in the methods used in APs and GPs and many scored the first 8 marks.
In (ii)B the main problem was distinguishing between the n^{th} term and the sum of the terms up to it. Those who used the latter got nowhere. The common error in the work using $5 \times (1.1)^{n-1}$ was to log it as $\log 5 \times (n-1)\log 1.1$.
Most good candidates chose the correct formula and scored the 4 marks.
Nearly all the candidates solved the given inequality and arrived at $n > 25.16$ but about a third left that as their answer and another third rounded to 25.